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E-Banhatti Indices of Chain Silicate Networks

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Abstract

Topological indices are numerical descriptors of chemical Structures. Silicates containing Silicon-oxygen compounds make up about 95 percent of Earth's Crust and upper mantle. In this work, we study first and second E-Banhatti indices , other indices of Chain Silicate Networks.

Keywords: Chain Silicate Networks, Polynomials, Sombor Index.

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1. Introduction

Let G be the simple, finite, connected graph. Let V(G) and E(G) be the vertex set and edge set of a graph G respectively [5]. Topological indices are numerical representation of chemical structures. They have wide range of applications in chemical graph theory[4]. The degree of vertex u denoted by d(u) is the number of vertices adjacent to the vertex u and the edge e is incident by the vertices u and v with edge e=uv. The degree of an edge in graph G is defined as d(e)=d(u)+d(v)-2.

The Banhatti degree of a vertex u in a graph G defined as [7],

$$B(u) = \frac{d(e)}{n - d(u)}$$

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The first and second E-Banhatti indices and their polynomials were defined by Kulli in [7] as,

$$EB_1(G) = \sum_{uv \in E(G)} [B(u) + B(v)]$$

$$EB_2(G) = \sum_{uv \in E(G)} B(u) \times B(v)$$

$$EB_1(G, x) = \sum_{uv \in E(G)} x^{[B(u)+B(v)]}$$

$$EB_2(G, x) = \sum_{uv \in E(G)} x^{B(u) \times B(v)}$$

The first and second hyper E-Banhatti indices and their polynomials were defined by Kulli in [7] as,

$$HEB_1(G) = \sum_{uv \in E(G)} [B(u) + B(v)]^2$$

$$HEB_2(G) = \sum_{uv \in E(G)} [B(u) \times B(v)]^2$$

$$HEB_1(G, x) = \sum_{uv \in E(G)} x^{[B(u)+B(v))]^2}$$

$$HEB_2(G, x) = \sum_{uv \in E(G)} x^{[B(u) \times B(v))]^2}$$

E-Banhatti Nirmala index , modified E-Banhatti Nirmala index of graph G and their exponential form were defined by Kulli in [8] as ,

$$EBN(G) = \sum_{uv \in E(G)} \sqrt{B(u) + B(v)}$$

$$m_{EBN(G)} = \sum_{uv \in E(G)} \frac{1}{\sqrt{B(u) + B(v)}}$$

$$EBN(G, x) = \sum_{uv \in E(G)} x^{\sqrt{B(u)+B(v)}}$$

$$m_{EBN(G,x)} = \sum_{uv \in E(G)} x^{\frac{1}{\sqrt{B(u) + B(v)}}}$$

The E-Banhatti Sombor index , modified E-Banhatti Sombor index of a graph G and their exponential form were defined by Kulli in [9] as,

$$EBS(G) = \sum_{uv \in E(G)} \sqrt{B(u)^2 + B(v)^2}$$

$$m_{EBS(G)} = \sum_{uv \in E(G)} \frac{1}{\sqrt{B(u)^2 + B(v)^2}}$$

$$EBS(G, x) = \sum_{uv \in E(G)} x^{\sqrt{B(u)^2+B(v)^2}}$$

$$m_{EBS(G,x)} = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{B(u)^2+B(v)^2}}}$$

The product connectivity E-Banhatti index,reciprocal product connectivity E-Banhatti index of a graph G and their polynomials defined by kulli in [10] as,

$$PEB(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{B(u)\times B(v)}}$$

$$RPEB(G) = \sum_{uv\in E(G)} \sqrt{B(u)\times B(v)}$$

$$PEB(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{\sqrt{B(u)\times B(v)}}}$$

$$RPEB(G,x) = \sum_{uv\in E(G)} x^{\sqrt{B(u)\times B(v)}}$$

The first and second modified E-Banhatti indices of a graph G and their polynomials defined by Kulli in [11] as,

$$m_{EB_1(G)} = \sum_{uv\in E(G)} \frac{1}{B(u)+B(v))}$$

$$m_{EB_2(G)} = \sum_{uv\in E(G)} \frac{1}{B(u)\times B(v)}$$

$$m_{EB_1(G,x)} = \sum_{uv\in E(G)} x^{\frac{1}{B(u)+B(v)}}$$

$$m_{EB_2(G,x)} = \sum_{uv\in E(G)} x^{\frac{1}{B(u)\times B(v)}}$$

Several graph indices were studied in chemical graph theory such as Wiener index [1] ,[2], Zagreb indices[3],Gourava indices etc[12].

2. Results

Silicates containing Silicon-oxygen compounds are abundant in Earth's Crust and upper mantle.The Chain silicate network is denoted by CS_n , see Figure 1. We have three types of edges in graph G of CS_n having 3n+1 vertices and 6n edges. They are,

$E_1 = \{uv \in E(CS_n)|d(u) = d(v) = 3\}, \qquad |E_1| = n+4$
$E_2 = \{uv \in E(CS_n)|d(u) = 3, d(v) = 6\}, \qquad |E_2| = 4n-2$
$E_3 = \{uv \in E(CS_n)|d(u) = d(v) = 6\}, \qquad |E_3| = n-2$

We have three types of Banhatti edges as follows,

$BE_1 = \{uv \in E(CS_n)|B(u) = B(v) = \frac{4}{3n-2}\}, \qquad |BE_1| = n+4$

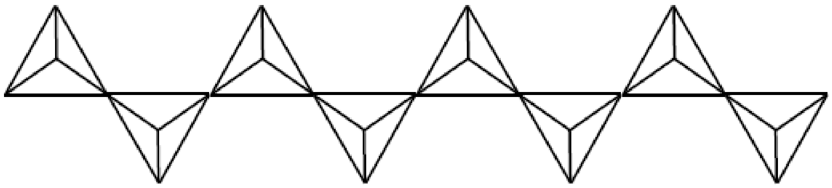

Figure 1: Chain Silicate Network

$$BE_2 = \{uv \in E(CS_n) | B(u) = \frac{7}{3n-2}, B(v) = \frac{7}{3n-5}\}, \qquad |BE_2| = 4n-2$$
$$BE_3 = \{uv \in E(CS_n) | B(u) = B(v) = \frac{10}{3n-5}\}, \qquad |BE_3| = n-2$$

Theorem 2.1. Let CS_n be a chain silicate network. Then

$$(i) EB_1(CS_n) = \frac{8(n+4)}{3n-2} + \frac{7(4n-2)(6n-7)}{(3n-2)(3n-5)} + \frac{20(n-2)}{3n-5}$$
$$(ii) EB_1(CS_n, x) = (n+4)x^{\left[\frac{8}{3n-2}\right]} + (4n-2)x^{\left[\frac{7(6n-7)}{(3n-2)(3n-5)}\right]} + (n-2)x^{\left[\frac{20}{(3n-5)}\right]}$$

Proof.

$$(i) EB_1(CS_n) = \sum_{uv \in E(CS_n)} [B(u) + B(v)]$$
$$= (n+4)\left[\frac{4}{3n-2} + \frac{4}{3n-2}\right] + (4n-2)\left[\frac{7}{3n-2} + \frac{7}{3n-5}\right] + (n-2)\left[\frac{10}{3n-5} + \frac{10}{3n-5}\right]$$

gives the required result after simplification.

$$(ii) EB_1(CS_n, x) = \sum_{uv \in E(CS_n)} x^{[B(u)+B(v)]}$$
$$= (n+4)x^{\left[\frac{4}{3n-2} + \frac{4}{3n-2}\right]} + (4n-2)x^{\left[\frac{7}{3n-2} + \frac{7}{3n-5}\right]} + (n-2)x^{\left[\frac{10}{3n-5} + \frac{10}{3n-5}\right]}$$

gives the required result after simplification.

□

Theorem 2.2. Let CS_n be a chain silicate network.Then

$$(i) EB_2(CS_n) = (n+4)\frac{16}{(3n-2)^2} + (4n-2)\frac{49}{(3n-2)(3n-5)} + (n-2)\frac{100}{(3n-5)^2}$$

$$(ii) EB_2(CS_n, x) = (n+4)x^{\left[\frac{16}{(3n-2)^2}\right]} + (4n-2)x^{\left[\frac{49}{(3n-2)(3n-5)}\right]} + (n-2)x^{\left[\frac{100}{(3n-5)^2}\right]}$$

Proof.

$$(i) EB_2(CS_n) = \sum_{uv\in E(CS_n)} B(u)\times B(v)$$
$$= (n+4)\left[\frac{4}{3n-2}\times\frac{4}{3n-2}\right] + (4n-2)\left[\frac{7}{3n-2}\times\frac{7}{3n-5}\right] + (n-2)\left[\frac{10}{3n-5}\times\frac{10}{3n-5}\right].$$

gives the required result after simplification.

$$(ii) EB_2(CS_n, x) = \sum_{uv\in E(CS_n)} x^{B(u)\times B(v)}$$
$$= (n+4)x^{\left[\frac{4}{3n-2}\times\frac{4}{3n-2}\right]} + (4n-2)x^{\left[\frac{7}{3n-2}\times\frac{7}{3n-5}\right]} + (n-2)x^{\left[\frac{10}{3n-5}\times\frac{10}{3n-5}\right]}$$

gives the required result after simplification.

□

Theorem 2.3. Let CS_n be a chain silicate network. Then

$$(i) HEB_1(CS_n) = (n+4)\frac{64}{(3n-2)^2} + (4n-2)\left[\frac{42n-49}{(3n-2)(3n-5)}\right]^2 + (n-2)\frac{400}{(3n-5)^2}$$

$$(ii) HEB_1(CS_n, x) = (n+4)x^{\left[\frac{64}{(3n-2)^2}\right]} + (4n-2)x^{\left[\frac{42n-49}{(3n-2)(3n-5)}\right]^2} + (n-2)x^{\left[\frac{400}{(3n-5)^2}\right]}$$

Proof.

$$(i) HEB_1(CS_n) = \sum_{uv\in E(CS_n)} [B(u)+B(v)]^2$$
$$= (n+4)\left[\frac{4}{3n-2}+\frac{4}{3n-2}\right]^2 + (4n-2)\left[\frac{7}{3n-2}+\frac{7}{3n-5}\right]^2 +$$
$$(n-2)\left[\frac{10}{3n-5}+\frac{10}{3n-5}\right]^2$$

gives the required result after simplification.

$$(ii) HEB_1(CS_n, x) = \sum_{uv\in E(CS_n)} x^{[B(u)+B(v)]^2}$$
$$= (n+4)x^{\left[\frac{4}{3n-2}+\frac{4}{3n-2}\right]^2} + (4n-2)x^{\left[\frac{7}{3n-2}+\frac{7}{3n-5}\right]^2} +$$
$$(n-2)x^{\left[\frac{10}{3n-5}+\frac{10}{3n-5}\right]^2}$$

gives the required result after simplification.

□

Theorem 2.4. Let CS_n be a chain silicate network. Then

$$(i)HEB_2(CS_n) = (n+4)\frac{256}{(3n-2)^2} + (4n-2)\left[\frac{2401}{(3n-2)^2(3n-5)^2}\right] + (n-2)\frac{10000}{(3n-5)^2}$$

$$(ii)HEB_2(CS_n, x) = (n+4)x^{\left[\frac{256}{(3n-2)^2}\right]} + (4n-2)x^{\left[\frac{2401}{(3n-2)^2(3n-5)^2}\right]} + (n-2)x^{\left[\frac{10000}{(3n-5)^2}\right]}$$

Proof.

$$(i)HEB_2(CS_n) = \sum_{uv \in E(CS_n)} [B(u) \times B(v)]^2$$
$$= (n+4)\left[\frac{4}{3n-2} \times \frac{4}{3n-2}\right]^2 + (4n-2)\left[\frac{7}{3n-2} \times \frac{7}{3n-5}\right]^2 + (n-2)\left[\frac{10}{3n-5} \times \frac{10}{3n-5}\right]^2$$

gives the required result after simplification.

$$(ii)HEB_2(CS_n, x) = \sum_{uv \in E(CS_n)} x^{[B(u) \times B(v)]^2}$$
$$= (n+4)x^{\left[\frac{4}{3n-2} \times \frac{4}{3n-2}\right]^2} + (4n-2)x^{\left[\frac{7}{3n-2} \times \frac{7}{3n-5}\right]^2} + (n-2)x^{\left[\frac{10}{3n-5} \times \frac{10}{3n-5}\right]^2}$$

gives the required result after simplification.

□

Theorem 2.5. Let CS_n be a chain silicate network. Then

$$(i)EBN(CS_n) = (n+4)\sqrt{\frac{8}{3n-2}} + (4n-2)\sqrt{\frac{42n-49}{(3n-2)(3n-5)}} + (n-2)\sqrt{\frac{20}{3n-5}}$$

$$(ii)EBN(CS_n, x) = (n+4)x^{\sqrt{\frac{8}{3n-2}}} + (4n-2)x^{\sqrt{\frac{42n-49}{(3n-2)(3n-5)}}} + (n-2)x^{\sqrt{\frac{20}{3n-5}}}$$

Proof.

$$(i)EBN(CS_n) = \sum_{uv \in E(CS_n)} \sqrt{B(u) + B(v)}$$
$$= (n+4)\sqrt{\frac{4}{3n-2} + \frac{4}{3n-2}} + (4n-2)\sqrt{\frac{7}{3n-2} + \frac{7}{3n-5}} + (n-2)\sqrt{\frac{10}{3n-5} + \frac{10}{3n-5}}$$

gives the required result after simplification.

$$(ii)EBN(CS_n, x) = \sum_{uv \in E(CS_n)} x^{\sqrt{B(u)+B(v)}}$$

$$= (n+4)x^{\sqrt{\frac{4}{3n-2}+\frac{4}{3n-2}}} + (4n-2)x^{\sqrt{\frac{7}{3n-2}+\frac{7}{3n-5}}} + (n-2)x^{\sqrt{\frac{10}{3n-5}+\frac{10}{3n-5}}}$$

gives the required result after simplification.

□

Theorem 2.6. Let CS_n be a chain silicate network. Then

$$(i)m_{\ EBN(CS_n)} = (n+4)\sqrt{\frac{3n-2}{8}} + (4n-2)\sqrt{\frac{(3n-2)(3n-5)}{42n-49}} + (n-2)\sqrt{\frac{3n-5}{20}}$$

$$(ii)m_{\ EBN(CS_n,x)} = (n+4)x^{\sqrt{\frac{3n-2}{8}}} + (4n-2)x^{\sqrt{\frac{(3n-2)(3n-5)}{42n-49}}} + (n-2)x^{\sqrt{\frac{3n-5}{20}}}$$

Proof.

$$(i)m_{\ EBN(CS_n)} = \sum_{uv \in E(CS_n)} \frac{1}{\sqrt{B(u)+B(v)}}$$

$$= (n+4)\left[\frac{4}{3n-2}+\frac{4}{3n-2}\right]^{-\frac{1}{2}} + (4n-2)\left[\frac{7}{3n-2}+\frac{7}{3n-5}\right]^{-\frac{1}{2}} + (n-2)\left[\frac{10}{3n-5}+\frac{10}{3n-5}\right]^{-\frac{1}{2}}$$

gives the required result after simplification.

$$(ii)m_{\ EBN(CS_n,x)} = \sum_{uv \in E(CS_n)} x^{\frac{1}{\sqrt{B(u)+B(v)}}}$$

$$= (n+4)x^{\left[\frac{4}{3n-2}+\frac{4}{3n-2}\right]^{-\frac{1}{2}}} + (4n-2)x^{\left[\frac{7}{3n-2}+\frac{7}{3n-5}\right]^{-\frac{1}{2}}} + (n-2)x^{\left[\frac{10}{3n-5}+\frac{10}{3n-5}\right]^{-\frac{1}{2}}}$$

gives the required result after simplification. □

Theorem 2.7. Let CS_n be a chain silicate network.Then

$$(i)EBS(CS_n) = (n+4)\left[\frac{4\sqrt{2}}{3n-2}\right] + (4n-2)\left[\frac{7\sqrt{(3n-5)^2+(3n-2)^2}}{(3n-2)(3n-5)}\right] + (n-2)\left[\frac{10\sqrt{2}}{(3n-5)}\right]$$

$$(ii)EBS(CS_n, x) = (n+4)x^{\left[\frac{4\sqrt{2}}{3n-2}\right]} + (4n-2)x^{\left[\frac{7\sqrt{(3n-5)^2+(3n-2)^2}}{(3n-2)(3n-5)}\right]} + (n-2)x^{\left[\frac{10\sqrt{2}}{(3n-5)}\right]}$$

Proof.

$$(i)EBS(CS_n) = \sum_{uv \in E(CS_n)} \sqrt{B(u)^2 + B(v)^2}$$
$$= (n+4)\sqrt{\left(\frac{4}{3n-2}\right)^2 + \left(\frac{4}{3n-2}\right)^2} + (4n-2)\sqrt{\left(\frac{7}{3n-2}\right)^2 + \left(\frac{7}{3n-5}\right)^2} +$$
$$(n-2)\sqrt{\left(\frac{10}{3n-5}\right)^2 + \left(\frac{10}{3n-5}\right)^2}$$

gives the required result after simplification.

$$(ii)EBS(CS_n, x) = \sum_{uv \in E(CS_n)} x^{\sqrt{B(u)^2 + B(v)^2}}$$
$$= (n+4)x^{\sqrt{\left(\frac{4}{3n-2}\right)^2 + \left(\frac{4}{3n-2}\right)^2}} + (4n-2)x^{\sqrt{\left(\frac{7}{3n-2}\right)^2 + \left(\frac{7}{3n-5}\right)^2}} +$$
$$(n-2)x^{\sqrt{\left(\frac{10}{3n-5}\right)^2 + \left(\frac{10}{3n-5}\right)^2}}$$

gives the required result after simplification.

□

Theorem 2.8. Let CS_n be a chain silicate network. Then

$$(i)m_{\ EBS(CS_n)} = (n+4)\left[\frac{3n-2}{4\sqrt{2}}\right] + \frac{(4n-2)}{7}\left[\frac{(3n-2)(3n-5)}{\sqrt{(3n-5)^2 + (3n-2)^2}}\right] + (n-2)\left[\frac{(3n-5)}{10\sqrt{2}}\right]$$
$$(ii)m_{\ EBS(CS_n, x)} = (n+4)x^{\left[\frac{3n-2}{4\sqrt{2}}\right]} + \frac{(4n-2)}{7}x^{\left[\frac{(3n-2)(3n-5)}{\sqrt{(3n-5)^2 + (3n-2)^2}}\right]} + (n-2)x^{\left[\frac{(3n-5)}{10\sqrt{2}}\right]}$$

Proof.

$$(i)m_{\ EBS(CS_n)} = \sum_{uv \in E(CS_n)} \frac{1}{\sqrt{B(u)^2 + B(v)^2}}$$
$$= (n+4)\left[\left(\frac{4}{3n-2}\right)^2 + \left(\frac{4}{3n-2}\right)^2\right]^{-\frac{1}{2}} + (4n-2)\left[\left(\frac{7}{3n-2}\right)^2 + \left(\frac{7}{3n-5}\right)^2\right]^{-\frac{1}{2}} +$$
$$(n-2)\left[\left(\frac{10}{3n-5}\right)^2 + \left(\frac{10}{3n-5}\right)^2\right]^{-\frac{1}{2}}$$

gives the required result after simplification.

$$(ii)m_{EBS(CS_n,x)} = \sum_{uv \in E(CS_n)} x^{\frac{1}{\sqrt{B(u)^2+B(v)^2}}}$$

$$= (n+4)x^{\left[\left(\frac{4}{3n-2}\right)^2+\left(\frac{4}{3n-2}\right)^2\right]^{-\frac{1}{2}}} + (4n-2)x^{\left[\left(\frac{7}{3n-2}\right)^2+\left(\frac{7}{3n-5}\right)^2\right]^{-\frac{1}{2}}} +$$

$$(n-2)x^{\left[\left(\frac{10}{3n-5}\right)^2+\left(\frac{10}{3n-5}\right)^2\right]^{-\frac{1}{2}}}$$

gives the required result after simplification. □

Theorem 2.9. Let CS_n be a chain silicate network. Then

$$(i)PEB(CS_n) = (n+4)\left[\frac{(3n-2)}{4}\right] + (4n-2)\left[\frac{\sqrt{(3n-2)(3n-5)}}{7}\right] + (n-2)\left[\frac{(3n-5)}{10}\right]$$

$$(ii)PEB(CS_n,x) = (n+4)x^{\left[\frac{(3n-2)}{4}\right]} + (4n-2)x^{\left[\frac{\sqrt{(3n-2)(3n-5)}}{7}\right]} + (n-2)x^{\left[\frac{(3n-5)}{10}\right]}$$

Proof.

$$(i)PEB(CS_n) = \sum_{uv \in E(CS_n)} \frac{1}{\sqrt{B(u)+B(v)}}$$

$$= (n+4)\left[\frac{4}{3n-2} + \frac{4}{3n-2}\right]^{-\frac{1}{2}} + (4n-2)\left[\frac{7}{3n-2} + \frac{7}{3n-5}\right]^{-\frac{1}{2}}$$

$$+ (n-2)\left[\frac{10}{3n-5} + \frac{10}{3n-5}\right]^{-\frac{1}{2}}$$

gives the required result after simplification.

$$(ii)PEB(CS_n,x) = \sum_{uv \in E(CS_n)} x^{\frac{1}{\sqrt{B(u)+B(v)}}}$$

$$= (n+4)x^{\left[\frac{4}{3n-2}+\frac{4}{3n-2}\right]^{-\frac{1}{2}}} + (4n-2)x^{\left[\frac{7}{3n-2}+\frac{7}{3n-5}\right]^{-\frac{1}{2}}} +$$

$$(n-2)x^{\left[\frac{10}{3n-5}+\frac{10}{3n-5}\right]^{-\frac{1}{2}}}$$

gives the required result after simplification. □

Theorem 2.10. Let CS_n be a chain silicate network. Then

$$(i) RPEB(CS_n) = (n+4)\left[\frac{4}{3n-2}\right] + (4n-2)\left[\frac{7}{\sqrt{(3n-2)(3n-5)}}\right] + (n-2)\left[\frac{10}{3n-5}\right]$$

$$(ii) RPEB(CS_n, x) = (n+4)x^{\left[\frac{4}{3n-2}\right]} + (4n-2)x^{\left[\frac{7}{\sqrt{(3n-2)(3n-5)}}\right]} + (n-2)x^{\left[\frac{10}{3n-5}\right]}$$

Proof.

$$\begin{aligned}(i) RPEB(CS_n) &= \sum_{uv \in E(CS_n)} \sqrt{B(u) \times B(v)} \\ &= (n+4)\sqrt{\frac{4}{3n-2} \times \frac{4}{3n-2}} + (4n-2)\sqrt{\frac{7}{3n-2} \times \frac{7}{3n-5}} + \\ &\quad (n-2)\sqrt{\frac{10}{3n-5} \times \frac{10}{3n-5}}\end{aligned}$$

gives the required result after simplification.

$$\begin{aligned}(ii) RPEB(CS_n, x) &= \sum_{uv \in E(CS_n)} x^{\sqrt{B(u) \times B(v)}} \\ &= (n+4)x^{\sqrt{\frac{4}{3n-2} \times \frac{4}{3n-2}}} + (4n-2)x^{\sqrt{\frac{7}{3n-2} \times \frac{7}{3n-5}}} + \\ &\quad (n-2)x^{\sqrt{\frac{10}{3n-5} \times \frac{10}{3n-5}}}\end{aligned}$$

gives the required result after simplification.

□

Theorem 2.11. Let CS_n be a chain silicate network. Then

$$(i) m_{EB_1(CS_n)} = \frac{(n+4)(3n-2)}{8} + (4n-2)\left[\frac{(3n-2)(3n-5)}{(42n-49)}\right] + (n-2)\left[\frac{3n-5}{20}\right]$$

$$(ii) m_{EB_1(CS_n, x)} = (n+4)x^{\left[\frac{(3n-2)}{8}\right]} + (4n-2)x^{\left[\frac{(3n-2)(3n-5)}{(42n-49)}\right]} + (n-2)x^{\left[\frac{3n-5}{20}\right]}$$

Proof.

$$\begin{aligned}(i) m_{EB_1(CS_n)} &= \sum_{uv \in E(CS_n)} \frac{1}{B(u) + B(v)} \\ &= (n+4)\left[\frac{1}{\frac{4}{3n-2} + \frac{4}{3n-2}}\right] + (4n-2)\left[\frac{1}{\frac{7}{3n-2} + \frac{7}{3n-5}}\right] + \\ &\quad (n-2)\left[\frac{1}{\frac{10}{3n-5} + \frac{10}{3n-5}}\right]\end{aligned}$$

gives the required result after simplification.

$$(ii) m_{EB_1(CS_n,x)} = \sum_{uv \in E(CS_n)} x^{\frac{1}{B(u)+B(v)}}$$

$$= (n+4)x^{\left[\frac{1}{\frac{4}{3n-2}+\frac{4}{3n-2}}\right]} + (4n-2)x^{\left[\frac{1}{\frac{7}{3n-2}+\frac{7}{3n-5}}\right]} + (n-2)x^{\left[\frac{1}{\frac{10}{3n-5}+\frac{10}{3n-5}}\right]}$$

gives the required result after simplification.

□

Theorem 2.12. Let CS_n be a chain silicate network. Then

$$(i) m_{EB_2(CS_n)} = (n+4)\left[\frac{(3n-2)^2}{16}\right] + (4n-2)\left[\frac{(3n-2)(3n-5)}{49}\right] + (n-2)\left[\frac{(3n-5)^2}{100}\right]$$

$$(ii) m_{EB_2(CS_n,x)} = (n+4)x^{\left[\frac{(3n-2)^2}{16}\right]} + (4n-2)x^{\left[\frac{(3n-2)(3n-5)}{49}\right]} + (n-2)x^{\left[\frac{(3n-5)^2}{100}\right]}$$

Proof.

$$(i) m_{EB_2(CS_n)} = \sum_{uv \in E(CS_n)} \frac{1}{B(u)\times B(v)}$$

$$= (n+4)\left[\frac{1}{\frac{4}{3n-2}\times\frac{4}{3n-2}}\right] + (4n-2)\left[\frac{1}{\frac{7}{3n-2}\times\frac{7}{3n-5}}\right] + (n-2)\left[\frac{1}{\frac{10}{3n-5}\times\frac{10}{3n-5}}\right]$$

gives the required result after simplification.

$$(ii) m_{EB_1(CS_n,x)} = \sum_{uv \in E(CS_n)} x^{\frac{1}{B(u)\times B(v)}}$$

$$= (n+4)x^{\left[\frac{1}{\frac{4}{3n-2}\times\frac{4}{3n-2}}\right]} + (4n-2)x^{\left[\frac{1}{\frac{7}{3n-2}\times\frac{7}{3n-5}}\right]} + (n-2)x^{\left[\frac{1}{\frac{10}{3n-5}\times\frac{10}{3n-5}}\right]}$$

gives the required result after simplification. □

Conclusions

In this work , we determined some E-Banhatti indices and their corresponding polynomials for chain silicate networks.

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